MMAT5010 Linear Analysis (2024-25): Homework 1
Deadline: 25 Jan 2025

Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard.

1. Let $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ be normed spaces. Now for each element $(x, y) \in X \oplus Y$ (the direct sum of X and Y) we put $\|(x, y)\|_\infty := \max(\|x\|_X, \|y\|_Y)$. Show that $(X \oplus Y, \|\cdot\|_\infty)$ is a Banach space if and only if X and Y both are Banach spaces.

2. Let (x_n) be a sequence in a normed space X.

   (a) Suppose that there is $0 < r < 1$ such that $\|x_n\| < r^n$ for all $n = 1, 2....$ Put $s_n := \sum_{k=1}^n x_k$. Show that if X is a Banach space, then $\sum_n x_n := \lim_n s_n$ exists in X.

   (b) Consider the finite sequence space $(c_{00}, \|\cdot\|_\infty)$. For each $n = 1, 2...$, let $x_n(k) = 1/2^n$ as $k = n$, otherwise, set $x_n(k) = 0$, i.e. $x_n := (0, ..., 0, 1/2^n, 0, ...) \in c_{00}$ at the n-th position is $1/2^n$. We keep the notation as Part (a). Show that $\lim_n s_n$ does not exist in c_{00}.

$* * *$ **End** $* * *$